UK Junior Mathematical Olympiad 2003

Organised by The United Kingdom Mathematics Trust

Tuesday 10th June 2003

RULES AND GUIDELINES : READ THESE INSTRUCTIONS CAREFULLY BEFORE STARTING

1. Time allowed: 2 hours.
2. **The use of calculators *and measuring instruments* is forbidden.**
3. All candidates must be in *School Year 8 or below* (England and Wales), *S2 or below* (Scotland), *School Year 9 or below* (Northern Ireland).
4. For questions in Section A *only the answer is required*. Enter each answer neatly in the relevant box on the Front Sheet. Do not hand in rough work.

   For questions in Section B you must give *full written solutions*, including clear mathematical explanations as to why your method is correct.

   Solutions must be written neatly on A4 paper. Sheets must be STAPLED together in the top left corner with the Front Sheet on top.

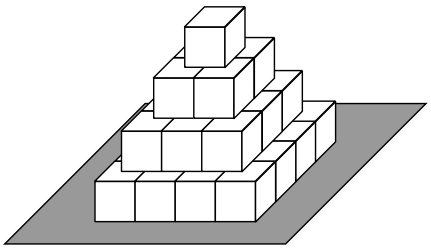
   Do not hand in rough work.
5. Questions A1-A10 are relatively short questions. Try to complete Section A within the first hour so as to allow at least one hour for Section B.
6. Questions B1-B6 are longer questions requiring *full written solutions*.
   This means that each answer must be accompanied by clear explanations and proofs.
   Work in rough first, then set out your final solution with clear explanations of each step.
7. These problems are meant to be challenging! Do not hurry. Try the earlier questions in each section first (they tend to be easier). Try to finish whole questions even if you can't do many. A good candidate will have done most of Section A and given solutions to at least two questions in Section B.
8. Answers must be FULLY SIMPLIFIED, and EXACT using symbols like π, fractions, or square roots if appropriate, but NOT decimal approximations.

DO NOT OPEN THE PAPER UNTIL INSTRUCTED BY THE INVIGILATOR TO DO SO!

Section A

A1 Calculate $\sqrt{4 + \sqrt{16 + \sqrt{81}}}$.

A2 The diagram shows a solid tower built from identical white cubes.
The outer surface of the tower, apart from where it is in contact with the table, is now painted red.
If the tower is then broken up into individual cubes, how many cubes will still be completely white?



A3 Sam starts to list, in ascending order, every positive integer which is *not* a factor of 720.
What is the tenth number on her list?

A4 In this addition, G, N and O represent different digits, none of which is zero.
What is the value of $G + N + O$?

$$\begin{array}{r} ON \\ ON \\ ON \\ +ON \\ \hline GO \end{array}$$

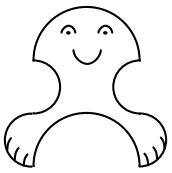
A5 Part of a bridge spans a river 35 metres wide. One third of the length of the bridge is on one side of the river and 20% of the length of the bridge is on the other side. How long is the bridge?

A6 Given a "starting" number, you double it and add 1, then divide the answer by 1 less than the starting number to get the "final" number.
If you start with 2, your final number is 5. If you start with 4, your final number is 3.
What starting number gives the final number 4?

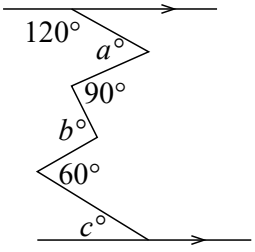
A7 What is the sum of the five numbers which must be placed in the empty cells of this magic square so that every row, every column and both diagonals all have the same total?

7		8
12		13

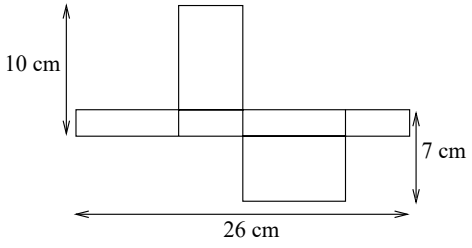
A8 The perimeter of this symmetrical creature is made from two semicircles of radius 4 cm and four semicircles of radius 2 cm.
Find the area enclosed by this perimeter.



A9 Find the value of $a + b + c$.



A10 This is the net of a cuboid. What is the volume of the cuboid?



Section B

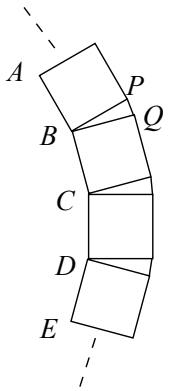
Your solutions to Section B will have a major effect on the JMO results. Concentrate on one or two questions first and then **write out full solutions** (not just brief 'answers').

B1 Find all of the ways in which 200 can be written in the form $p + q^2 + r^3$ where p, q and r are prime numbers.

B2 Five children, boys Vince, Will, and Zac and girls Xenia and Yvonne, sit at a round table. They come from five different cities, Aberdeen, Belfast, Cardiff, Durham and Edinburgh. The child from Aberdeen sits between Zac and the child from Edinburgh. Neither of the two girls is sitting next to Will. Vince sits between Yvonne and the child from Durham. Zac writes to the child from Cardiff.

Find, giving reasons, where each child comes from.

B3 The diagram shows part of a ring of squares and triangles around a regular polygon which has vertices A, B, C, D, E, ….



(a) Suppose the polygon $ABCDE\ldots$ has 10 sides. Calculate the sizes of $\angle ABC$ and $\angle BQP$.

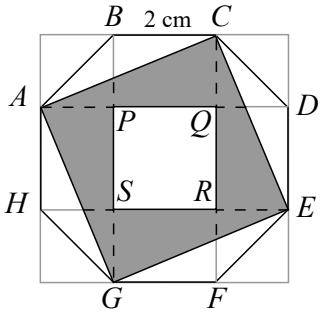
(b) Suppose the regular polygon has N sides.
Find the value of the ratio of the size of $\angle ABC$ to the size of $\angle BQP$.

B4 (a) Find the sum of all positive three-digit integers each of whose digits is either 2 or 3.

(b) Find the sum of all positive six-digit integers each of whose digits is either 2 or 3, giving your answer as the product of prime numbers.

B5 The diagram shows two squares, $ACEG$ and $PQRS$, inside a regular octagon $ABCDEFGH$ which has sides of length 2 cm.

What fraction of the entire octagon is shaded?



B6 12 is a 2-digit number such that the number '1' formed by the first digit is divisible by 1 and the number '12' formed by the first two digits is divisible by 2.

(a) How many 3-digit numbers '*abc*' are there, where a, b, c are the digits 1, 2, 3 in some order, such that '*a*' is divisible by 1, '*ab*' is divisible by 2 and '*abc*' is divisible by 3?

(b) How many 4-digit numbers '*abcd*' are there, where a, b, c, d are the digits 1, 2, 3, 4 in some order, such that '*a*' is divisible by 1, '*ab*' is divisible by 2, '*abc*' is divisible by 3 and '*abcd*' is divisible by 4?

(c) How many 5-digit numbers '*abcde*' are there, where a, b, c, d, e are the digits 1,2,3,4,5 in some order, such that '*a*' is divisible by 1, '*ab*' is divisible by 2, '*abc*' is divisible by 3, '*abcd*' is divisible by 4 and '*abcde*' is divisible by 5?

(d) How many 6-digit numbers '*abcdef*' are there, where a, b, c, d, e, f are the digits 1, 2, 3, 4, 5, 6 in some order, such that '*a*' is divisible by 1, '*ab*' is divisible by 2, '*abc*' is divisible by 3, '*abcd*' is divisible by 4, '*abcde*' is divisible by 5 and '*abcdef*' is divisible by 6?